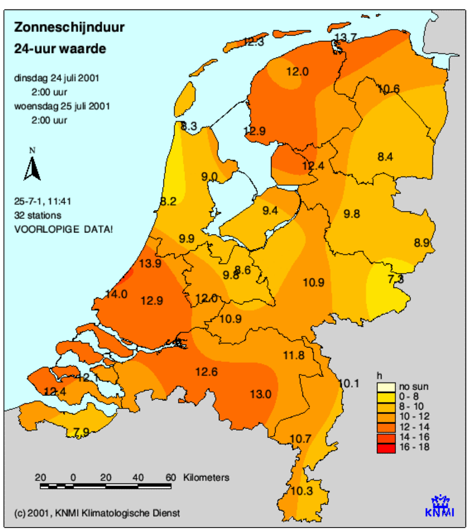
This is the first article in the Handbook series, based on the chapter entitled ‘Mathematical Literacy’ by **Eva Jablonka** from *Freie Universität Berlin*. **Kees Hoogland** discusses the key issues from this chapter and suggests what the trends indicated by Eva Jablonka could mean for mathematics education in the Netherlands.

Mathematical Literacy and Numeracy

Introduction

When I talk to people about numeracy and mathematical literacy, it is clear that these concepts are associated with many different images, notions and connotations. I have found the same when reading articles about numeracy and mathematical literacy. The first comment made is almost always that there is no unequivocal definition, that people perceive these terms in all kinds of different ways and that these terms are used, rightly or wrongly, to convey all kinds of things.

I personally find the use of real images a powerful way to initiate discussion about definitions. I have given examples throughout this article.



In various countries many different words are used to convey the concepts of numeracy and mathematical literacy.

English: Numeracy, Mathematical Literacy.
American: Mathematical Literacy, Mathematical Proficiency
German: Mathematische Grundbildung, Mathematische Kompetenz.
French: La culture mathématique.
Dutch: Gecijferdheid, wiskundige geletterdheid.
Spanish: La formación matemática.

The definition I personally prefer is that of Jeff Evans given in the book entitled *‘Adults’ Mathematical Thinking and Emotion’*, a definition that revolves very much around the individual. “Numeracy is the ability to process, interpret and communicate numerical, quantitative, spatial, statistical, even mathematical, information, in ways that are appropriate for a variety of contexts, and that will enable a typical member of the culture or subculture to participate effectively in activities that they value.” (Evans, 2000)

The chapter by Eva Jablonka deals extensively with a number of different definitions of mathematical literacy. These are discussed a little further on. At the end of this article, I shall look at what the noted trends could or should mean for mathematics education in the Netherlands.

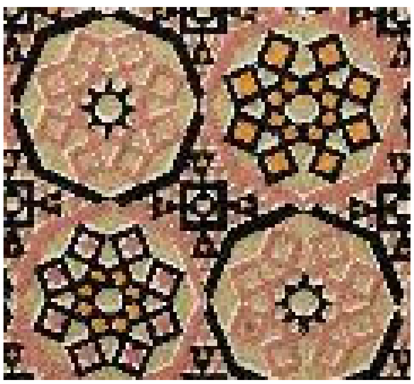
A Brief Look Back in Time

In 1996 the *International Handbook of Mathematics Education* first appeared, an extensive and sound overview of how things stood in mathematics education around the world: what the thinking was, what research was being conducted and what the main developments were in mathematics education.

In the opening chapter of the book, the international developments in primary and lower secondary education are summarized in one sentence by Mogens Niss: ‘The goals of primary and lower secondary mathematics education have been broadened considerably so as to encompass the essential aspects of numeracy and “mathematical literacy” in society.’ (Niss, 1996). It becomes clear that this book is retrospective when you see that the concept of numeracy is listed in the index only once as appearing in a chapter about adult education (FitzSimons, 1996). The term mathematical literacy was not yet in use and did not feature in the index.

In 2003, the *Second International Handbook of Mathematics Education* was published. Judging by the structure, the content and the index, it is clear that interest in numeracy and mathematical literacy has increased. The book

begins with a section on mathematics education and policy, which includes chapters with titles such as ‘Mathematics, mathematics education and economic conditions’, ‘Is mathematics for all?’, ‘Lifelong mathematics education’, ‘Mathematics education in international and global contexts’ and a chapter on ‘Mathematical literacy’.
This shows that over the past decade, the concepts of numeracy and mathematical literacy have gained wider recognition and increasing influence in the determination of the goals and structure of mathematics education. In this article I intend to concentrate in more detail on Eva Jablonka’s chapter entitled ‘Mathematical Literacy’.



Mathematical Literacy

Eva Jablonka’s title ‘Mathematical Literacy’ is a deliberate and carefully-considered choice on her part, since she believes that ‘numeracy’ refers too much to numerical aspects, whereas ‘mathematical literacy’ denotes both the much wider approach of mathematics as well as ‘literacy’, which has a connotation of well-informed individuals who have had a sound education.
Mathematical literacy has been defined in a wide variety of ways over the past few years and one thing everyone more or less has come to agree on is that mathematical literacy cannot be defined in terms of mathematical knowledge. Mathematical literacy is in fact mainly about the functional aspect of mathematical knowledge. It is about individual competencies to use mathematical knowledge in a practical, functional way; mathematical literacy in order to …. or mathematical literacy for …

Eva Jablonka classifies the different approaches into five categories. She justifiably concludes that the differences in approach are directly linked with the goals that are pursued in mathematics education in individual countries. She reviews the following approaches:
Mathematical literacy for developing human capital
Mathematical literacy for cultural identity
Mathematical literacy for social change
Mathematical literacy for environmental awareness
Mathematical literacy for evaluating mathematics
The second, third and fourth approach have relatively few followers, in the Western world at least. The first approach is fairly prevalent because it is one that is used by a large number of international comparative studies. The last approach is Eva Jablonka’s preferred approach and it is therefore given the most attention in the chapter.

Mathematical Literacy for Developing Human Capital
This approach focuses on equipping people with the mathematical tools to interpret and organize their day-to-day lives. The idea is that all kinds of problems can be modelled with mathematics and can subsequently be solved with mathematical techniques. These are problems from day-to-day life as well as problems in the work place and problems at a global level. The maths needed for this is seen as universal.

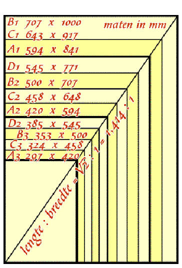
This is also the approach used by the OECD, which defines mathematical literacy as follows: ‘The capacity to identify, to understand and to engage in mathematics and make well-founded judgements about the role that mathematics plays, as needed for an individual’s current and future private life, occupational life, social life with peers and relatives, and life as a constructive, concerned, and reflective citizen.’ (OECD,1999)

The international comparative study PISA also works with this definition. From this perspective

maths is not culture-bound and is free from value judgements and therefore globally comparable. According to the OECD there is an automatic causality between learning maths and raising living standards, increasing democratic empowerment and development of economic potential, etc.
However, one wonders whether all the problems in the world can be resolved simply through the use of mathematical tools. The OECD used almost the same type of argument back in the 60s when New Maths was introduced.

Mathematical Literacy for Cultural Identity
This approach is prevalent in the developing countries, where there is a great discrepancy between the maths the pupils have to learn at school and the maths they use in their everyday lives and which they use to solve everyday problems. Indeed, this approach serves as argument to give this informal maths a more important place in the curriculum. This informal maths can be seen as a form of mathematical literacy. This approach has been referred to as ‘ethnomathematics’ for some time now; the mathematics of the market and the work place. It is an approach that is particularly strongly rooted in South America and Africa.

It is interesting that from this perspective, there is opposition to the translation of curricula and textbooks from other countries and cultures, because it has the negative psychological effect on teachers and pupils that maths is something that comes from outside as opposed to something that is inextricably linked with their own world, something that does not fit in with their cultural identity. This approach is therefore at odds with the view that everywhere in the world mathematical literacy has to involve the same kind of mathematics.



Mathematical Literacy for Social Change
This is the critical pedagogics perspective, i.e. mathematical literacy is a capacity to view reality differently and change it.

Mathematical literacy needs to lead primarily to critical citizens. This approach strongly criticizes school mathematics, which only leads to continued inequality in knowledge, social class and sex.
Maths should focus mainly on the critical consideration of socially and politically meaningful issues, especially if they are associated with statistics. Other subjects that should be tackled in class as part of the mathematical literacy programme include energy prices, child mortality figures and life expectancy figures.
This approach is somewhat reminiscent of that of the critical teachers of the 70s who saw education as a way towards equipping pupils with the ability to hold their own in society and life in general.

Mathematical Literacy for Environmental Awareness
The view that mathematics provided a useful and beneficial contribution to industrialization and science and therefore to the improvement of living conditions and the level of welfare in the population has wide support all over the world.
However, those who support the ‘mathematical literacy for environmental awareness’ approach find that mathematics has also contributed to military technology and to the highly polluting mass industries in low-wage countries. They would like to see examples of how mathematics can contribute to the analysis of global environmental problems so as to make all kinds of people, but particularly also mathematicians, much more aware of this role of mathematics. Taking this even further, supporters even suggest that there should be a totally different kind of mathematics, in which the emphasis is more on creativity and on an open way of thinking about all kinds of developments, a kind of environmental mathematics or green mathematics so to speak.
This approach has much in common with the previous approach, although it seems to be more of an 80s and 90s variant.

Mathematical Literacy for Evaluating Mathematics
Mathematics is used explicitly and implicitly throughout society:
- in the school subject of Maths;

- in all kinds of political discussions involving figures;
- in models and graphical representations;
- in technology.

In this approach, mathematical literacy should include learning to identify and being able to critically evaluate mathematics and the role that mathematics plays. This implies a rather passive role at first, seeing and identifying mathematics. However, this is seldom found in school mathematics, which mostly revolves around doing sums.
This form of mathematical literacy learning therefore requires a great deal of discussion and dialogue during maths lessons. The way to organize this into effective teaching programmes in schools is by no means clear yet.
Eva Jablonka gives a few examples of explicit uses of maths in society for illustration. The main uses include:
- the use of condensed measures and indexes: there is an increasing tendency to bunch all kinds of phenomena into one figure: inflation, consumer confidence, productivity, share prices and so on. The more complex the composition of the figures, the greater the chances that the only thing that is talked about is the end result of the figures and not the way they were compiled, whereas there could be a great deal of discussion about which factors contributed to these indicators and in what ratio. A mathematically literate adult should be aware of the danger of fully relying on figures that have been compiled like this. These kinds of indicators can be valuable, but only if they are accompanied with the relevant social or political background;
- the use of models: it is fairly easy to take a few empirical data and make up a model that fits with these data. Automation makes this process ever easier. And once a model has been made up, it can easily take on a life of its own. Critically evaluating these models should therefore be part of mathematical literacy.

For instance: the Verhulst model of population growth (1845) is well known:

$$P(n + l) = k \,.\, P(n) - c \,.\, P(n)^2$$

This model is built on a sound theoretical foundation, which was described in the articles in which the model was presented. The model was used for a number of years as a good way to predict population growth in the United States. It does, however, have an asymptomatic upper limit, making it seem as if the population is gently growing towards a maximum number. Uncritical use of this model led to the supposition that this would also be true in reality.

The following model is from an English study into the likelihood of criminals re-offending:

$$S = 31 - A - C + 75 \sqrt{\left(\frac{g}{F + 5} + K \right)}$$

Where A is the age, C is the number of detentions, g the number of condemnations and F the number of years after the initial crime. K is a constant determined by the type of crime and can be looked up in a list.

This formula does no harm in theory (neither does it do much good), but there is reason for concern when such a formula is used uncritically by probation officers or by policy-makers within the justice system. Indeed there seems to be no acceptable theoretical background for the form of this model, however well the formula may fit with a certain set of data. Mathematical literacy should include the awareness that a model should only be used if the form of the model itself and of the formulas of the model is deemed acceptable on the basis of theoretical considerations.

Jablonka's Conclusions

At the end of the chapter, Eva Jablonka enunciates a number of conclusions. From those conclusions, I have selected those that I believe are most relevant to mathematics education at upper secondary school level.
- There are many different ways to analyse the relationship between the maths taught in schools and that used outside school and the various definitions of mathematical literacy revolve around this relationship because they relate to an individual's capacity to use the mathematics that should be learnt at school.
- The ability to understand the mathematical aspects of everyday situations and to make a judgement about them forms an important part of mathematical literacy.

- The capacity to evaluate critically is not in itself mathematical by nature and neither is it a result of a high level of mathematical thinking.
- Introducing critical discussions implies bringing in a great deal of classroom discussion about the uses of mathematics. This will ultimately lead to well-informed citizens dealing with mathematics outside of school in a new way.

I have not much more to add to this and therefore continue this article with an assessment of the impact the reported trends may have on today's mathematics education.

International Consequences

Over the coming ten years the following discussions about the organization of mathematics education are to be expected world-wide. For the purpose of this article I shall, however, limit myself to upper secondary education.
The mathematics curricula will need to strike a balance between elements related to mathematical literacy, to learning mathematical techniques and to getting to grips with the academic mathematical way of thinking. Furthermore increasing automation of all kinds of mathematical techniques and the application of mathematics in other subjects also come into play. Then there is also the trend towards competencies and towards a problem-oriented approach to problems and topics that transcends subject boundaries. It seems that the thinking around mathematical literacy dovetails more closely with this trend than the thinking around more traditional applications of the maths curriculum.
These discussions could gather momentum once the results are published of a number of international comparative studies into the standards of mathematics and mathematical literacy. By sheer coincidence the four-yearly TIMSS study, the three-yearly PISA study and the IALS/ALL study (which includes a numeracy component) are all due to be repeated in 2003. Most of the results will be published in 2004 and will no doubt give rise to heated discussion internationally. All these studies undeniably test for a large part the standard of mathematical literacy and in fact to a far lesser extent the standard of technical mathematics skills.

Consequences for Mathematics Education in the Netherlands

The discussion about what role mathematical literacy can and should play in the various mathematics curricula in upper secondary education will also need to take place in the Netherlands in the next few years. Here are some suggestions.

Upper secondary level: HAVO and VWO Mathematics for All: Maths A
In the early 80s, Maths A was an innovating, refreshing programme, enabling large groups of pupils to become aware of the role mathematics plays in realistic situations. Even then there were goals related to mathematical literacy: critically evaluating statistics and graphical representations and solving problems from everyday life using mathematical techniques. Since then there have been no substantial changes to the subject for twenty years and it has lost its freshness and meaning to a large extent as a result. Is manual two-dimensional linear programming, for instance, the answer to the mathematical literacy related problems young people encounter in society? The latest plans of the Dutch Ministry of Education (2003) for the maths curricula in the second stage of HAVO and VWO propose to scrap Maths A1 from HAVO, and VWO will follow suit very quickly. Furthermore a hybrid form of Maths AB is being proposed. Surely it is not too complicated to devise a modern curriculum that is challenging for all A pupils and that fits into the mathematical literacy agenda (A1), and to conclude it with a school examination that is properly quality controlled.
The next thing that should be looked into is what mathematical models are useful in economic activities and what maths is needed to understand and implement automated mathematical techniques (A2).

Lower Secondary Level: VMBO, HAVO and VWO

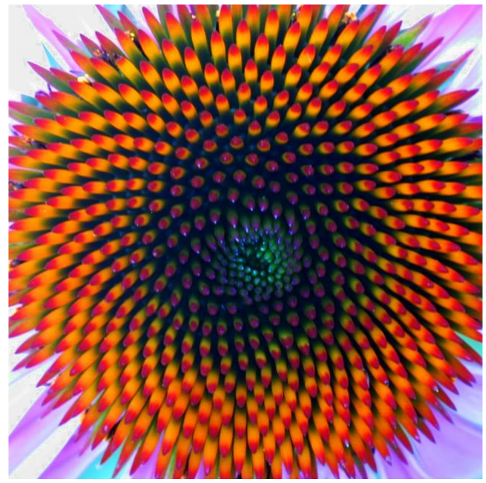
The ideas of W12-16 that led to the new lower secondary education programme in 1993 were no doubt influenced by all kinds of ideas on numeracy and mathematical literacy. The most poignant example of this was the area of Information Processing and Statistics. Remarkably, this component was increasingly marginalized in subsequent editions of the textbooks. All that remains is: sum, answer, checking the answer on the CD. This is understandable to an extent since this is no different in the final examinations. This will have to change dramatically again in the coming years if mathematical literacy becomes an integral part of the curriculum.

Vocational Education: VMBO

Within VMBO, which is still fairly young, there are all kinds of opportunities in the next few years to make mathematical literacy part of maths education. For the theoretical programme, the combined theoretical and vocational programme and the middle-management programme in the technology and agriculture sectors it is important to look carefully at what mathematical techniques are of use to the pupils. However, the pupils also need a good dose of mathematical literacy if they are to manage in today's complex society. And it may be important to make a clear distinction between techniques and mathematical literacy within the programme. The current programme seems to have a little bit of everything: every comparison has to be put in a realistic context and every critical evaluation of a realistic situation has to be accompanied with a sum.

Then there is also a large group of pupils in the basic vocational programme, some receiving learning support and some not, some with apprenticeships and some not. The current maths curriculum has simply nothing to offer these pupils; no attention is paid to numeracy, and techniques are reduced to tricks or questions most pupils can answer very easily. Asking these pupils to sit centrally devised final written exams in mathematics and marking these against centrally determined standards is a meaningless charade that is certainly of no benefit to the pupils or their achievement levels. It is precisely these kinds of pupils who benefit from a good foundation in numeracy, to enable them to manage in situations involving money, medicines, decoding information and so on.

For pupils in an apprenticeship it has been made compulsory for their training to include language, computer skills and a vocational subject. This shows that the decision-making for VMBO is not done by people who know about the international discussions about literacy. Otherwise it would surely simply have said that these pupils must have a sound and relevant foundation in literacy and numeracy, tailored to the sector they are interested in and tailored to their apprenticeship training programme.



Classroom Activities in Numeracy / Mathematical Literacy

1. The website www.gecijferdheid.nl comprises a variety of PowerPoint presentations with a collection of photographs of numeracy-related situations. These pictures could be used to initiate a discussion among the pupils about the quantitative aspects of the world around us. The pupils could add their own photographs.

2. Ask the pupils to think of things they did the previous day that involved numbers, from the time they woke up to the time they went to bed. First working by themselves, then coming together in pairs and then bringing everything together to the whole group. Don't be surprised if the pupils come up with dozens or even more than a hundred instances. It will help pupils become aware of the huge variety of activities that involve numeracy. The effect could be that they begin to be more aware of how numbers feature in the world around them.

3. Ask the pupils to spend two weeks gathering examples of real activities where they had to use their skills with numbers. The aim is to create a personal collage of numeracy-related situations that mean something to the individual pupils. They can then use the collage to say something about themselves.

What all these activities have in common is that there is no question of sums, but that they are in fact aimed at initiating discussion about numeracy / mathematical literacy.

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Websites

www.oecd.org : including information about PISA

www.ets.org/all : information about the IALS/ALL study

www.woodrow.org/need/national_numeracy_network.html : information about the National Numeracy Network (USA)

www.dfes.gov.uk/numeracy : information about the National Numeracy Strategy (UK)

www.gecijferdheid.nl : Kees Hoogland's Dutch-language website on numeracy and mathematical literacy (under construction)

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